Title:
Construction of Volume Parametric Model Based on Quadrilateral Subdivision of Surface

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Introduction:
The volume parametric model does not require format conversion or mesh generation and can be directly used for IGA (isogeometric analysis), offering the potential for seamless integration of CAD and CAE. The representation of the volumetric parametric model must be a zero-genus two-variable tensor product surface or a three-variable tensor product volume. Therefore, complex multi-genus geometric models or non-two-variable tensor product surfaces need to be divided into zero-genus convex quadrilaterals patches. The study of convex decomposition of simple polygons expressed by straight lines is relatively mature, such as the Q-Morph method [1], which performs splitting or merging on triangular meshes to form quadrilateral meshes. There is still significant research potential in the area of curved quadrilateral subdivision. Chen et al. [2] performed curved quadrilateral subdivision on planar curved polygonal regions based on the geometric features of curves, but this method requires manually added connecting lines. Damian et al. [3] proposed a method for subdividing a given polygon into multiple sub-polygons. However, the resulting number of sub-polygons is relatively large, leading to a significant increase in computational complexity for subsequent calculations. Zhang et al. [4] proposed a template- and field-guided subdivision method to generate planar regions with a coarse quadrilateral layout. These methods are unable to perform full curved quadrilateral subdivision on freeform surfaces, failing to meet the modeling requirements for volume parametric models.

Based on planar quadrilateral subdivision and the concept of conformal mapping, this paper proposes a method for constructing volume parametric models using surface quadrilateral subdivision, aiming to minimize the number of generated curved quadrilaterals. First, a least-squares conformal parameterization method is used to flatten the spatial surface into a two-dimensional plane and extract the contours. Next, a planar quadrilateral subdivision algorithm is applied to divide the plane into curved quadrilateral polygons. Based on the mapping relationship between the surface and the plane, the plane is then mapped back onto the spatial surface. Finally, the parametric model construction is completed by incorporating feature operations. Based on the method proposed in this paper, a fully quadrilateral subdivision of a multi genus triangular mesh surface is performed, and a volume parametric model is generated based on the subdivision results. The generated examples demonstrate that the method for constructing complex volume parameterized models based on surface subdivision algorithm is robust and effective, providing strong support for volume parametric model construction.

Surface Flattening Based on Conformal Mapping:

The least-squares conformal parameterization method [5] is a type of free boundary parameterization. In this paper, this method is used to flatten triangular mesh surfaces into a plane. The second figure of Fig 1 shows the result of the conformal parameterization flattening of a multi-genus hemisphere.

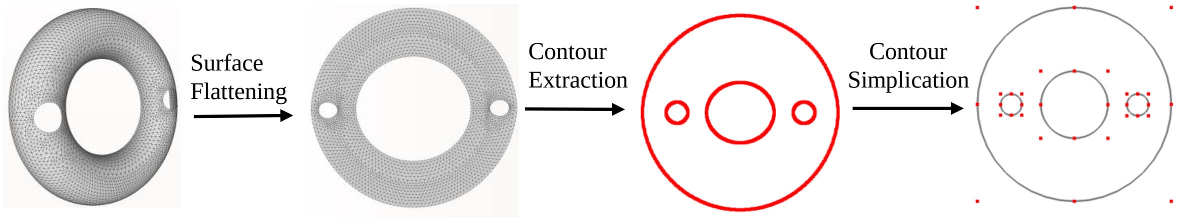

Fig. 1: Surface flattening.

After flattening the original surface, it is necessary to extract the geometric contours of the flattened plane and convert them into boundary representations using NURBS splines for subsequent quadrilateral subdivision. The third figure of Fig 1 represents the extracted spline contours. The extracted model contours contain a large number of spline curves, which complicates the subsequent full quadrilateral subdivision. Therefore, it is necessary to simplify the contours, representing the model outline with as few spline curvs as possible. The right figure of Fig 1 shows the result after simplifying the model contours.

Planar Quadrilateral Subdivision:

After completing the surface flattening, contour extraction, and simplification, the next step is to perform quadrilateral subdivision on the resulting planar geometric contours of the model. The planar polygonal quadrilateral subdivision algorithm proposed in [2] still requires manual addition of inner and outer contour connecting lines for geometries with genus. In this paper, we improve upon this algorithm by automating the generation of connecting lines, thereby achieving fully automated curved quadrilateral subdivision.

First, the inner and outer contours of the model are input, and an improved ray-casting method is used to determine the inclusion relationships of the contours, constructing the corresponding geometric domain inclusion tree. The results of the planar geometric domain set and the constructed geometric domain inclusion tree are shown in Fig 2. Then, the root node of the geometric domain inclusion tree and the contour lines of its child nodes are extracted, and the contour lines are sorted in a clockwise direction. The endpoints on the contours of the root node and its child nodes are obtained sequentially, and the connecting lines are constructed. Besides, intersection checks between line segments are performed to exclude connecting lines that intersect with the contours. This process is repeated iteratively until the leaf nodes of the inclusion tree are reached, resulting in the identification of all possible connecting lines.

To ensure the quality of the resulting patches in the subsequent meshing process, the generated connecting lines must not form an excessively small angle with their adjacent contours, meaning that sharp corners should be avoided. An angle check is required, and the weight-based method is adopted to filter the generated connecting lines. As shown in Fig 3, AB and AC are the generated connecting lines. Taking the connecting line AB as an example, the angles θ_1 and θ_2 between AB and the adjacent boundary of the inner contour it connects to is calculated. The smaller the difference $\Delta_1 = \|\theta_1 - \theta_2\|$, the closer the angles are to each other, indicating a position nearer to the ideal configuration. Next,

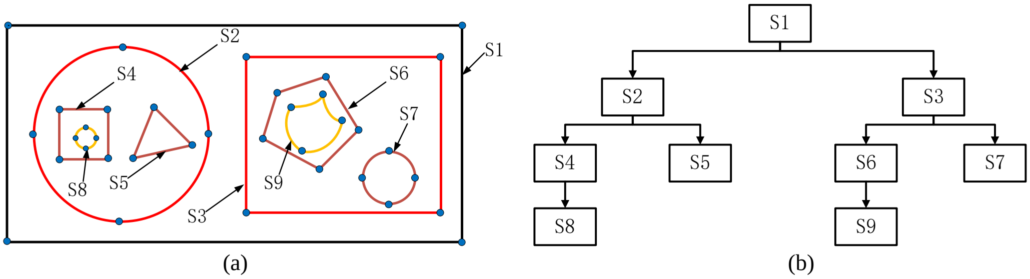

Fig. 2: Geometric domain inclusion tree.

the angles θ_3 and θ_4 between the connecting line AB and the adjacent boundaries of the outer contour it connects to are calculated. The difference $\Delta_2 = \|\theta_3 - \theta_4\|$ is determined, and the sum of the two angle differences $w_1 = \Delta_1 + \Delta_2$ is the weight. Similarly, the angles $\alpha_i(i = 1, ..., 4)$ corresponding to the connecting line AC and their differences θ_3 and θ_4 are calculated to obtain the weight w_2. The weights w_1 and w_2 are compared, with smaller weights indicating a more optimal position for the connecting line. In the same manner, the weights of all connecting lines generated between the outer and inner contours are calculated, resulting in a set of weights $w = \{w_1, w_2, , ..., w_n\}$. All connecting lines are sorted based on their weights, and the line with the smallest weight is selected as the final result for the inner and outer contours.

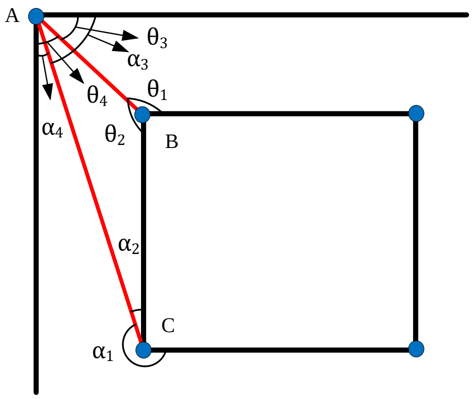

Fig. 3: Connection line filtering.

Similarly, using a multi-genus circular plane as an example, curved quadrilateral subdivision is performed. Fig 4 shows the subdivision process.

Spatial Mapping:

After completing the fully automated quadrilateral subdivision on the plane and obtaining the results, the geometric contours of each quadrilateral patch are extracted. Based on the mapping relationship between the surface and the plane, the quadrilateral contours are mapped back onto the surface, and

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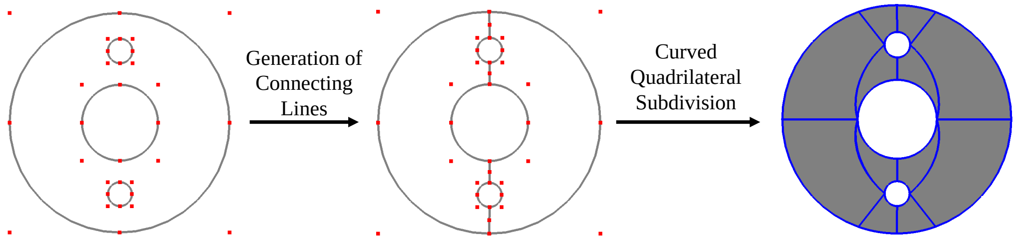

Fig. 4: Curved quadrilateral subdivision process.

then a spline surface is obtained through surface fitting. Fig 5 illustrates the process of generating the spatial spline surface.

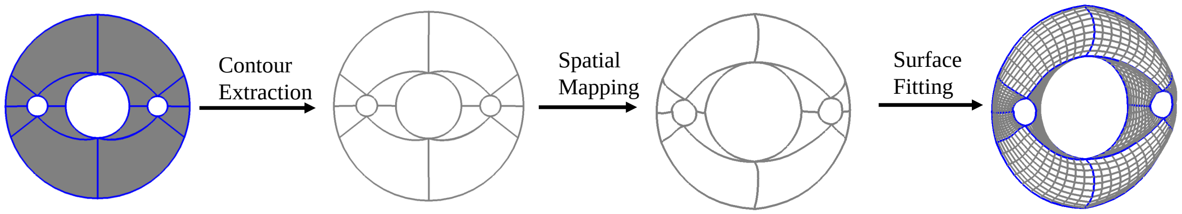

Fig. 5: Spatial spline surface generation process.

Examples:

Using the method proposed in this paper, parametric models of a four-way pipe and a spherical shell with genus 8 were constructed. After completing the surface full quadrilateral subdivision, a corresponding volume parametric model is generated using a sweeping operation. Then, a mirror operation is used to obtain the other part of the volume parametric model, and feature merging is applied to generate the complete volume parametric model. Finally, the generated volume parametric model is imported into Paraview® to visualize the Jacobian distribution and evaluate the model quality. Fig 6 (a) and (b) shows the complete volume parametric models of a four-way pipe and a spherical shell obtained through the mirror operation respectively, while Fig 6 (c) and (d) displays the Jacobian value distribution of the models. As seen in the figure, the Jacobian values are all greater than 0, which meets the requirements for isogeometric analysis.

Conclusions:

This paper presents a method for constructing volume parametric models based on surface quadrilateral subdivision. The model examples demonstrate that the proposed method is robust and effective, capable of performing full quadrilateral subdivision on genus g surfaces, thereby enabling the construction of volume parametric models that include genus g surfaces. However, during the process of establishing the mapping relationship between the plane and the surface, surface reconstruction may introduce modeling expression errors. Therefore, developing a better spline surface conformal mapping method will be the focus of the next step.

Proceedings of CAD'25, Shenzhen, China, June 23-25, 2025, 250-254

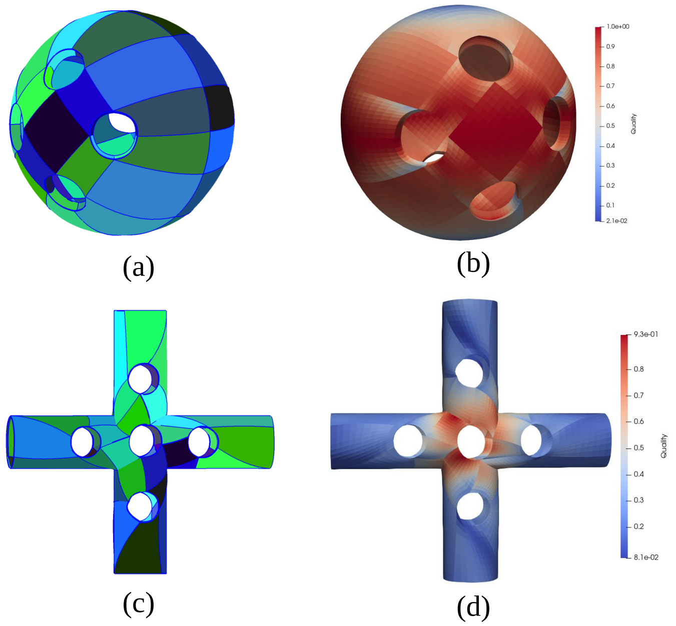

Fig. 6: Examples of volume parametric model.

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