

<u>Title:</u>

Multi-Objective Optimization Algorithms for Generative Design: Performance Evaluation in Custom Dental Abutment Design

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Introduction:

Generative design (GD) is an innovative design methodology that leverages computational algorithms to autonomously create, analyze, and optimize design solutions [4]. Unlike traditional approaches that rely on manual iteration and designer expertise, GD explores extensive solution spaces to discover innovative configurations that may not be apparent to a human designer. By defining design parameters and optimization objectives, generative algorithms systematically generate a diverse range of feasible solutions, evaluate their performance based on predefined criteria, and iteratively refine them to achieve optimal design outcomes. This makes GD especially effective in addressing complex, performancedriven design challenges where conflicting objectives must be balanced. This has made GD particularly valuable in industries such as aerospace, automotive, architecture, and healthcare, where highly optimized and efficient solutions are essential. When integrated with modern CAD tools, GD becomes even more powerful by enabling parametric modeling, real-time visualization, and streamlined iteration, which significantly accelerates the design process and reduces development costs. At the core, GD relies on advanced optimization algorithms, particularly multi-objective optimization algorithms (MOOAs), which are essential for solving problems with competing goals. Most existing studies on MOOAs in GD use synthetic benchmarks, limiting insights into their real-world applicability [3], [4]. This paper evaluates three MOOAs, MOEA/D, NSGA-III, and RBFMOpt, in the practical context of custom dental abutments requiring patient-specific geometry, biomechanical stability, and manufacturing feasibility. We analyze algorithm performance concerning geometric objectives, convergence, computational efficiency, and precision to advance GD for industrial-scale customization. The paper is organized as follows: Section 2 introduces selected algorithms; Section 3 details the methodology; Section 4 presents results; Section 5 provides a discussion, and Section 6 concludes with key findings and future directions.

MOOAs in GD:

To provide a representative and methodologically balanced comparison, we selected three algorithms: Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D), Non-Dominated Sorting Genetic Algorithm III (NSGA-III), and Radial Basis Function Multi-Objective Optimization (RBFMOpt). Each is drawn from one of the three primary categories of multi-objective optimization strategies: decomposition-based, Pareto-based, and surrogate-assisted [9]. In addition to their methodological diversity, all three algorithms are readily available to designers through Grasshopper plugins (a visual programming environment integrated with Rhino3D, commonly used for parametric and generative design), making them accessible for practical use in CAD-integrated generative design workflows. This combination of methodological diversity and tool availability ensures a relevant and meaningful performance comparison. MOEA/D approaches multi-objective optimization by decomposing the problem into numerous singleobjective subproblems [11]. Each subproblem represents a specific trade-off between objectives, determined by a weight vector. The combined solutions of these subproblems construct the Pareto front. a set of optimal trade-offs where no objective can be improved without negatively affecting at least one other objective. For implementation on a parametric model, the first step involves defining the model's parameters as design variables with specified ranges. The optimization process begins when the algorithm generates a set of weight vectors that represent the relative importance of each objective in the first generation of variations. A population of initial solutions is created by randomly assigning values to the parameters within defined ranges. Each design is evaluated based on a scalarized fitness value, which combines the weighted objectives. Solutions are then grouped based on their weight vectors, allowing interaction and influence among nearby subproblems. Genetic operators, such as mutation and crossover, are used to evolve the population iteratively with a trend of replacing weaker solutions with stronger ones. In contrast, NSGA-III approaches the task holistically, treating the objectives as a unified whole. The NSGA-III builds on the principles of non-dominated sorting and incorporates a reference-point-based strategy to address high-dimensional objective spaces effectively [8]. Each design is evaluated based on multiple objectives, and the population is ranked into fronts using a non-dominated sorting mechanism. The first front contains the non-dominated solutions, while subsequent fronts consist of solutions dominated by the previous ones. NSGA-III enhances diversity preservation by introducing a set of predefined or adaptive reference points that guide the selection of solutions, ensuring an even distribution across the Pareto front. Genetic operators, such as crossover and mutation, are applied to evolve the population, with diversity being maintained by aligning solutions to the closest reference points. While both MOEA/D and NSGA-III rely on evaluation of solution directly using the true objective functions, **RBFMOpt** [5] uses surrogate functions to approximate the objective space, significantly reducing computational costs. First, design variable and ranges are defined, followed by generating an initial sample of designs that form the training data for the surrogate model. This surrogate model predicts the performance of new candidate solutions, focusing on promising regions of the design space, thereby reducing the number of expensive function evaluations. In each iteration, the most promising candidates are selected for true evaluation, and the resulting data are used to update the surrogate model.

Methodology:

Building a parametric model is essential as it allows for flexibility, automation, and efficient optimization by enabling systematic modifications to design parameters without manually reconstructing the geometry. In this study, a parametric model for custom dental abutments was developed using Rhino3D in combination with Grasshopper, leveraging visual programming and multiple GD plugins to facilitate design exploration and optimization. In general, the design of an abutment includes an implant connection interface (ICS), designed for a specific dental implant system within the bone, as well as two personalized segments: the transgingival segment (TS) and the prosthetic connection segment (PCS) (Fig. 1). TS extends through the gingival tissue, creating a seal that accommodates the unique emergence profile determined by individual gingival contours and implant placement. The PCS supports the final crown or bridge by transferring functional loads to the implant. Its geometry is influenced by factors such as implant angulation, distance to the opposing jaw, proximity to adjacent teeth, and the material selected for the restoration. Since the personalization of the abutment is carried out on the TS and PCS, the primary focus of the parametric model in this study is on these two segments. The transgingival segment is shaped based on four prosthetic surfaces (Fig. 1, right): distal (D), mesial (M), buccal (B), and oral (O) [1].

For each prosthetic surface, a single point is defined at the intersection of the surface centerline with the tissue, forming a reference point for gingival margin curve approximation. Consequently, the area between the ICS and the gingival margin defines the TS in this parametric model. Each point is described with two parameters, distance from ICS axis (Y_d , Y_m , X_o , X_b) and height distance (Z_d , Z_m , Z_o , Z_b) from the abutment interface, meaning TS contains eight modifiable parameters (Fig. 2). The shape of the PCS follows the gingival margin and is created as an extrusion of a curve offset toward the abutment axis from the gingival margin. The direction and length of the extrusion are determined by the position of the PCS axis, which is adjusted to accommodate the implant's placement angle and the position of

the opposing jaw. Therefore, the PCS is described using three parameters: angulation around the X-axis (A_x) , angulation around the Y-axis (A_y) , and the height of the segment (H) (Fig. 2).



Fig. 1: Dental abutment (left); Prosthetics and tissue surfaces (right).



Fig. 2: Abutment design parameters.

Together, these eleven parameters serve as the input variables to which GD algorithms were applied. GD algorithms were implemented using Opossum (MOEA/D and RBFMOpt) and Tunny (NSGA-III) Grasshopper plugins. Since the algorithms are designed to optimize objective functions (either by minimizing or maximizing them), specific control points were integrated into the patient's jaw scan (captured using an intraoral scanner), with each point representing a distinct objective. The objectives are: O1 - minimizing distance from point D on TS to jaw scan distal point, O2 - minimizing distance from point B on TS to jaw scan buccal point, O3 - minimizing distance from point M on TS to jaw scan mesial point, O4 - minimizing distance from point O on TS to jaw scan oral point and O5 - minimizing distance of abutment's PCS apex to opposite jaw influenced by angulation and height of PCS. Each algorithm is then compared based on simulation time, error per objective, and convergence rate [3]. Simulation time is crucial for practical design workflows and real-time feedback, while error per objective quantifies how accurately each solution aligns with the patient's anatomical requirements and indicates the algorithm's convergence behavior. The simulations were conducted for each algorithm under two scenarios, 500 and 1,500 design solutions, following recommendations from previous studies [10]. The simulations were performed on a workstation equipped with an AMD Ryzen 7 5700G CPU, 32 GB of RAM. and an NVIDIA RTX 4060 Ti GPU.

Results and discussion:

Tab. 1 presents a comparison of simulation time and minimal error per objective for each algorithm, highlighting their computational efficiency and accuracy.

Algorithm (solutions)	Time [min]	MIN error O1 [mm]	MIN error O2 [mm]	MIN error O3 [mm]	MIN error O4 [mm]	MIN error O5 [mm]
MOAE/D (500)	04:40	0,385	0,222	0,596	0,254	1,316
MOAE/D (1500)	10:08	0,475	0,523	0,645	0,176	1,054
NSGA-III (500)	09:30	0,497	0,030	0,041	0,479	0,263
NSGA-III (1500)	26:28	0,378	0,042	0,029	0,073	0,091
RBFMOpt (500)	05:40	0,371	0,046	0,044	0,039	0,100
RBFMOpt (1500)	45:20	0,37	0,027	0,000	0,027	0,002

Tab. 1: Simulation time and minimal errors per objective.

The comparison of the three algorithms, RBFMOpt, NSGA-III, and MOEA/D, reveals distinct strengths and weaknesses across different numbers of design solutions. According to Tab. 1, in the 500-solution session, MOEA/D completes the simulation the fastest, while NSGA-III takes the longest. This variation in simulation time is closely linked to the computational complexity of each algorithm. MOEA/D's decomposition-based approach allows for rapid solution processing, at a cost of lower accuracy, as it struggles to fully explore the Pareto front within a limited number of iterations [11]. In contrast, NSGA-III, which relies on non-dominated sorting and reference-point distribution, requires more computational effort to ensure convergence across objectives, leading to a longer runtime [5]. When comparing simulation time, MOEA/D, delivers the fastest but least accurate results across the defined objectives. NSGA-III achieves better solutions but still exhibits significant deviations in three out of five objectives, indicating that 500 solutions are insufficient for full convergence. This is particularly evident when results are compared to the 1,500-solution session, where NSGA-III shows substantial improvements across all objectives, whereas MOEA/D exhibits minimal progress, reinforcing the idea that the decomposition approach and weight vector distribution limit its performance. The increase in simulation time for NSGA-III is thus justified, allowing the algorithm to better refine solutions and more effectively distribute solutions across the Pareto front [7]. RBFMOpt consistently achieves the fastest convergence and the highest accuracy, exhibiting the smallest deviations across all objectives when the solution count is relatively low (500), a direct result of its surrogate modeling approach. By leveraging radial basis functions, RBFMOpt efficiently explores high-potential regions of the design space, minimizing the number of evaluations and achieving superior solutions with fewer evaluations [6]. As the number of solutions increases to 1,500, RBFMOpt continues to improve by minimizing objective errors outperforming both MOEA/D and NSGA-III. This improvement, however, comes at a steep increase of simulation duration (from 05:40 to 45:20 min), highlighting a key trade-off between computational efficiency and accuracy.

Fig. 3 visualizes the objective errors for the final top five solutions (S1-S5), illustrating the distribution of errors across different objectives and providing insights into the stability, precision, and trade-offs between accuracy and computational cost for each algorithm. Upon examining the diagram, it is evident that MOEA/D exhibits significant deviations in solutions across the Pareto front in the 500solution session. This is particularly pronounced in O1, O2, O4, and O5, where the error deviation within the selected solutions remains higher. The largest discrepancies are observed in O5, which also has the highest errors overall. This suggests that MOEA/D struggles to maintain consistency in the distribution of solutions, likely due to its decomposition approach. If the weight vectors are not well distributed or properly tuned, the algorithm may fail to generate a balanced set of trade-offs, leading to uneven solution guality across the Pareto front [7]. In contrast, NSGA-III demonstrates more stable solutions, as the differences between the maximum and minimum errors across objectives are smaller. This indicates that NSGA-III, even in the 500-solution session, is able to maintain better consistency across its recommended solutions, ensuring that extracted Pareto-optimal solutions are closer to one another in terms of accuracy [2]. RBFMOpt provides the most stable solutions, with minimal differences in errors across all objectives among the top five extracted solutions. This result suggests that the surrogate approach of RBFMOpt allows for precise refinement of solutions, consistently narrowing down promising regions of the solution space [10].



Fig. 3: Errors per objective after 500 solutions.

It is also important to note that O5 is directly influenced by three parameters, making it the most complex objective with the highest number of possible parameter combinations. This complexity increases the challenge for algorithms to efficiently converge on an optimal solution, particularly those like MOEA/D that are more sensitive to parameter distributions and require extensive tuning.

The abutments shown in Fig. 4 visually reflect the previously identified objective errors, with the most pronounced differences observed in the PCS (O5), as confirmed by the results in Tab. 1 and Fig. 3. The variation in PCS geometry highlights the discrepancies in how each algorithm optimizes this objective, further emphasizing the challenges associated with its complexity. Additionally, RBFMOpt is the only algorithm that satisfies the functional requirement of ensuring that the TS maintains continuous contact with the gingival margin on all reference surfaces.



Fig. 4: Generated abutment by each algorithm in session with 500 solutions (MOAE/D – left; NSGA-III – middle; RBFMOpt – right).

Conclusions:

This study compared the performance of three MOOAs, MOEA/D, NSGA-III, and RBFMOpt, applied to the generative design of custom dental abutments. The results indicate that each algorithm exhibits unique strengths and limitations, with trade-offs between accuracy, computational efficiency, and solution diversity, depending on the optimization scenario. MOEA/D, while computationally efficient, exhibited slower convergence and higher deviations of objective errors, particularly in high-dimensional objectives like O5. NSGA-III, despite requiring longer processing times, significantly improved its performance as the number of solutions increased, demonstrating the benefits of reference-point-based diversity preservation. RBFMOpt consistently demonstrated the fastest convergence rate and achieved the lowest objective errors across all evaluations. The study also raises the question of whether RBFMOpt's higher computational burden is always necessary to achieve acceptable solutions. In time-sensitive design workflows or computationally constrained environments, early-stage convergence may be sufficient, eliminating the need for extended simulations. Conversely, in cases requiring high precision or complex interactions between multiple parameters, increasing the number of solutions may be justified despite the longer simulation time. Comparison of the top five solutions per algorithm reveals that, from a user perspective, particularly for those with limited experience in interpreting optimization results, RBFMOpt offers the most reliable and least error-prone solutions. All five of its best solutions exhibit minimal variation (0.061–0.153 mm), meaning that any recommended outcome from the Pareto front is likely to be an acceptable final choice. This consistency makes RBFMOpt particularly well-suited for practical industrial applications, where minimizing decision-making complexity is beneficial. However, the study is subject to certain limitations. The evaluation was conducted using only three optimization algorithms, which, while representative of different methodological categories, does not encompass the full range of available approaches in generative design. Additionally, the study was performed using a single set of computational tools, specifically Rhino3D and Grasshopper, along with their respective optimization plugins. As different software environments and solver implementations can influence performance, further research should expand the analysis to a broader range of algorithms and tools to ensure more generalized conclusions. Future work could explore hybrid optimization strategies, combining the efficiency of surrogate-assisted modeling** with the diversity preservation of reference-point-based approaches, ensuring optimal trade-offs between computational cost, solution accuracy, and practical usability in generative design workflows.

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