Title:
Isogeometric Shape Optimization of Unstructured Spline-Based Thin Shell Structures

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Introduction:
Structural shape optimization is optimizing the geometric shape of engineering structures to obtain better mechanical performance [5]. The traditional finite element methods have been widely utilized to solve structural responses in the shape optimization process. As well known, the discretized nodes of the geometric model are normally treated as the design variables, and this strategy may lead to unwanted shapes. Isogeometric analysis, proposed by Hughes, aims to bridge the Computer-Aided Design (CAD) and Finite Element Analysis (FEA) by using the same geometric models. The spline functions used to represent geometric models in CAD are employed as the shape functions of analysis models in FEA. Consequently, the sophisticated mesh generation can be tactfully avoided. In isogeometric shape optimization, the control points can be regarded as the design variables, and the optimized shape can be exactly preserved without a fitting step.

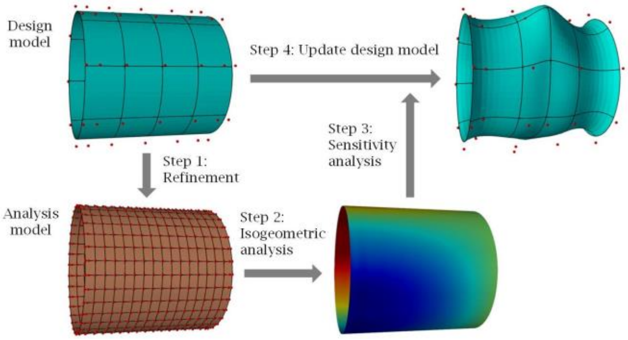

Fig. 1: The process of isogeometric shape optimization by using a multi-level model.

Due to the rectangular topology structure of Non-Uniform Rational B-Splines (NURBS), complex geometric models will be represented by multiple NURBS patches, whose domain may be trimmed [2,3]. The trimming and stitching features will introduce extra work to isogeometric analysis, e.g., coupling of adjacent patches, and integration of trimmed elements. Such geometric models are also difficult to be used for shape updating and control, since the relations between patches as well as single patch's control points need to be considered uniformly as design parameters, which will complicate the sensitivity analysis. So this paper developed a novel isogeometric shape optimization method for optimizing thin shell structures, in which the geometric models are represented by unstructured splines with multiple extraordinary points [6,8-10]. The introduction of extraordinary point enables the model to accommodate arbitrary topology shape features and greatly decreases the labors needed for geometry fixing. The Kirchhoff-Love shell theory is used to solve structural response. A multi-level model is developed to satisfy the different requirements of design and analysis, as shown in Fig. 1.

Isogeometric Shape Optimization:

Problem Definition

The structural shape optimization can be generally described as

$$\begin{aligned} &\text{find} && \mathbf{h} \in \mathbb{R}^{n_d} \\ &\min && f(\mathbf{h}) \\ &\text{s.t.} && c_i(\mathbf{h}) = 0, \quad \forall i \in \mathcal{E} \\ & && c_i(\mathbf{h}) \leq 0, \quad \forall i \in \mathcal{I} \end{aligned} \tag{2.1}$$

in which $\mathbf{h} = [h_1, h_2, \ldots, h_{nd}]^T$ is the design vector, nd is the number of design variables, f is the objective function, c_i is the constraint, $\mathcal{E}$ and $\mathcal{I}$ denote the equality and inequality constraint sets. The objective function can be represented as the function of design variables **h** and field variables **u**, namely,

$$f := f(\mathbf{h}, \mathbf{u}(\mathbf{h})). \tag{2.2}$$

For linear elastic problems, the relations between $\mathbf{h}$ and $\mathbf{u}$ is normally rewritten as

$$\mathbf{K}(\mathbf{h})\mathbf{u} = \mathbf{F}(\mathbf{h}), \tag{2.3}$$

where **K** is the stiffness matrix and **F** is the external vector.

Note that the geometric model can be precisely refined in IGA, we construct a multi-level model for shape optimization with a coarse-level model for geometric design and a dense-level model for numerical analysis. Let **P** and **Q** be the control points of the coarse and dense models, respectively. There is a linear mapping between **P** and **Q**, written as **Q** = **MP**. The matrix **M** can be obtained by using the classical knot insertion algorithm. Correspondingly, the derivative of any function (•) with respect to the control points **P** can be given by

$$\frac{\partial(\bullet)}{\partial \mathbf{P}} = \frac{\partial(\bullet)}{\partial \mathbf{Q}} \frac{\partial \mathbf{Q}}{\partial \mathbf{P}} = \frac{\partial(\bullet)}{\partial \mathbf{Q}} \mathbf{M}. \tag{2.4}$$

The sensitivity analysis is crucial in gradient-based optimization. An analytical adjoint-based method is utilized to compute sensitivity. The derivative of the objective function with respect to the design variables is calculated as

$$\frac{df}{dh_i} = \frac{\partial f}{\partial h_i} + \frac{\partial f}{\partial \mathbf{u}} \frac{d\mathbf{u}}{dh_i}, \quad i = 1, \ldots, n_d. \tag{2.5}$$

The derivate $d\mathbf{u}/dh_i$ can be recovered from Eqn. (2.3). Then the above equation can be rewritten as

$$\frac{df}{dh_i} = \frac{\partial f}{\partial h_i} + \frac{\partial f}{\partial \mathbf{u}} \mathbf{K}^{-1} \left(\frac{\partial \mathbf{F}}{\partial h_i} - \frac{\partial \mathbf{K}}{\partial h_i} \mathbf{u} \right), \quad i = 1, \ldots, n_d. \tag{2.6}$$

Introducing an adjoint solution $\mathbf{u}^*$, with

$$\mathbf{K}\mathbf{u}^* = \left(\partial f / \partial \mathbf{u} \right)^T, \tag{2.7}$$

to Eqn. (2.6), we can find

$$\frac{df}{dh_i} = \frac{\partial f}{\partial h_i} + \mathbf{u}^* \cdot \left(\frac{\partial \mathbf{F}}{\partial h_i} - \frac{\partial \mathbf{K}}{\partial h_i} \mathbf{u} \right), \quad i = 1, \ldots, n_d. \tag{2.8}$$

It can be found that the adjoint solution is solved once for each sensitivity analysis. The whole process is more efficient for problems with fewer constraints. Note that the sensitivity of the design variable can be generally transformed to that of control points. Combining Eqns. (2.4) and (2.8), leads to

$$\frac{df}{dh_i}=\frac{\partial f}{\partial h_i}+\left(\mathbf{u}^*\cdot\frac{\partial \mathbf{F}}{\partial \mathbf{Q}}-\mathbf{u}^*\cdot\frac{\partial \mathbf{Ku}}{\partial \mathbf{Q}}\right)\mathbf{M}\frac{\partial \mathbf{P}}{\partial h_i},\quad i=1,\ldots,n_d. \tag{2.9}$$

The total energy of a linear elastic system can be defined as

$$\mathcal{W}(\mathbf{u},\mathbf{u}^*,\mathbf{Q})=\mathcal{W}_{\text{ext}}(\mathbf{u}^*,\mathbf{Q})+\mathcal{W}_{\text{int}}(\mathbf{u},\mathbf{u}^*,\mathbf{Q})=\mathbf{u}^*\cdot\mathbf{F}-\mathbf{u}^*\cdot\mathbf{Ku}. \tag{2.10}$$

Then, the derivative in Eqn. (2.8) is simplified by

$$\frac{df}{dh_i}=\frac{\partial f}{\partial h_i}+\frac{\partial \mathcal{W}}{\partial \mathbf{Q}}\mathbf{M}\frac{\partial \mathbf{P}}{\partial h_i},\quad i=1,\ldots,n_d, \tag{2.11}$$

in which the term $\partial\mathbf{P}/\partial h_i$ can be obtained in the construction of the design model. The remaining unknown term is $\partial\mathcal{W}/\partial\mathbf{Q}$, which is related to the structural responses.

Structural Response and Sensitivity Analysis

The classical Kirchhoff-Love theory is employed to compute the structural response of shell models. In the Kirchhoff-Love shell theory, only three displacement DOFs are used, and C^1 continuity between adjacent elements should be satisfied. To achieve the C^1-continuity requirement, a D-patch framework [7] combined with smoothing matrices, truncation, and Bézier extraction is used to construct C^1-continuity function space over the unstructured mesh. Let $\bar{\mathbf{x}}$ be the middle surface of a shell structure defined by parametric coordinates (ξ,η), then the shell position can be represented by

$$\mathbf{x}(\xi,\eta,\zeta)=\bar{\mathbf{x}}(\xi,\eta)+\zeta a_3(\xi,\eta),\quad \zeta\in[-t/2,t/2] \tag{2.12}$$

where ζ is the parametric coordinate along the thickness direction, t indicates the thickness of the shell, a_3 is the unit normal vector.

The displacement of the shell structure, under linear elastic assumption can be described by the displacement of the middle surface and interpolated by spline basis functions, as

$$\mathbf{u}^e(\xi,\eta)=\sum_{i=0}^{n}R_i^e(\xi,\eta)\hat{\mathbf{u}}_i^e \tag{2.13}$$

in which $\hat{\mathbf{u}}_i^e$ denotes the displacement of the i-th control point, R_i^e indicates the non-zero spline function associated with the element e. The stiffness matrix and external vector are derived as

$$\mathbf{K}^e=\int_{\bar{\Omega}^e}\left(\mathbf{B}_m^T\mathbf{D}_0\mathbf{B}_m+\mathbf{B}_b^T\mathbf{D}_2\mathbf{B}_b\right)\left|\mathbf{J}\right|d\bar{\Omega}^e,\quad \mathbf{F}^e=\int_{\bar{\Omega}^e}\left(\mathbf{R}^e\right)^T\bar{\mathbf{b}}\left|\mathbf{J}\right|d\bar{\Omega}^e. \tag{2.14}$$

Readers can refer to [1,4] for a detailed derivation and explanation of the above equation. The derivative is given in Eqn. (2.11) can be computed by

$$\frac{\partial \mathcal{W}}{\partial \mathbf{Q}}=\frac{\partial \mathcal{W}_{\text{ext}}}{\partial \mathbf{Q}}+\frac{\partial \mathcal{W}_{\text{int}}}{\partial \mathbf{Q}}=\mathbf{u}^e\cdot\frac{\partial \mathbf{F}^e}{\partial q_{aj}^e}-\mathbf{u}^e\cdot\frac{\partial \mathbf{K}^e}{\partial q_{aj}^e}\mathbf{u}^e \tag{2.15}$$

Inserting Eqn. (2.14) into Eqn. (2.15), the derivative $\partial\mathcal{W}/\partial\mathbf{Q}$ for sensitivity analysis can be obtained.

<u>Numerical Examples:</u>

In this example, an L-shaped shell model with a central hole is taken as the design domain for shape optimization. As shown in Fig. 2(a), the top edge is fixed, and the bottom edge is subjected to a uniform line load F = 1N. For the material properties, Young's modulus $E = 6.825\times10^7$Pa, Poisson ratio ν = 0.3. The thickness of the shell takes t = 0.04m. It can be observed that there are several extraordinary points in the design model. To satisfy the C^1-continuity requirement for the Kirchhoff-Love shell, the analysis model S_{A0}, as given in Fig. 2(b), is constructed from the design model S_{D0} by using the D-patch scheme. Then, the initial analysis model is refined twice to obtain the dense models S_{A1} and S_{A2}. The geometric shape of the four surfaces is the same with each other. The number of control points is 104, 176, 720, and 2960. Three multi-level schemes are designed as follows

- Scheme #1: Taking S_{D0} as the design model and S_{A2} as the analysis model.

- Scheme #2: Taking S_{A0} as the design model and S_{A2} as the analysis model.
- Scheme #3: Taking S_{A1} as the design model and S_{A2} as the analysis model.

In each scheme, the two-layer boundary control points are constrained and the coordinates of central control points are treated as the design variables.

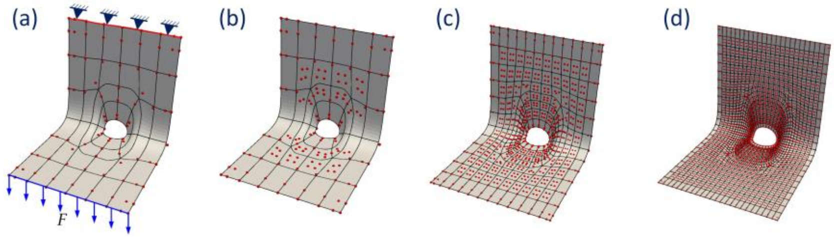

Fig. 2: The L-shaped shell model with a central hole. (a) Design model S_{D0} with boundary conditions, (b) analysis model S_{A0} and the refined analysis model (c) S_{A1}, (d) S_{A2}.

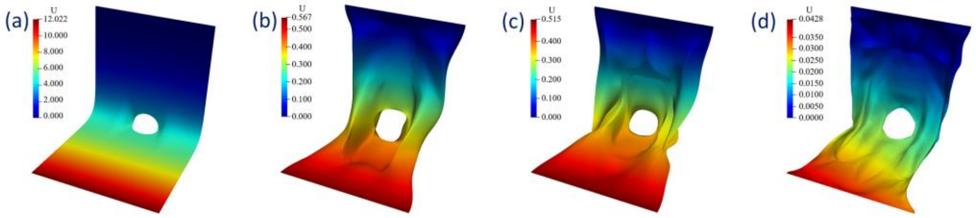

Fig. 3: The isogeometric analysis results for (a) initial design model with compliance 182.601 J, (b) optimized shape using scheme #1, with compliance 8.7043 J, (c) optimized shape using scheme #2, with compliance 7.2267 J, and (d) optimized shape using scheme #4, with compliance 1.0399 J.

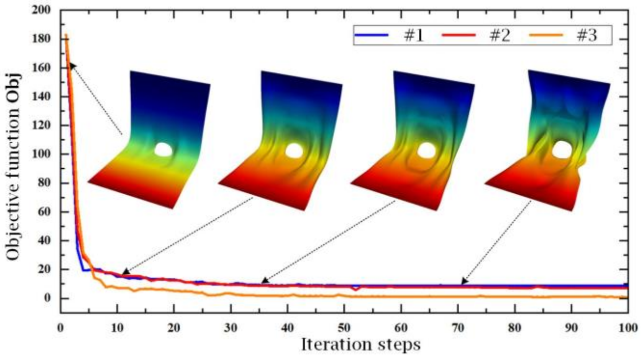

Fig. 4: The convergence curve for the isogeometric shape optimization of the L-shaped shell by using three schemes #1, #2, and #3.

Figure 3 illustrates the isogeometric analysis results of the initial model and the optimized models. It can be found that the compliance values of the three optimized models are 8.7043J, 7.2267J and 1.0399J, which are much lower than that of the initial model 182.601J. However, the geometric shape of the three optimized models is different. More control points in the design model will lead to more details on the optimized model. Although scheme #3 presents the lowest compliance, there are plenty of wrinkles on the model and the surface quality is poor. The optimized model obtained by using schemes #2 and #3 is relatively reasonable and is smoother than that of #1. The convergence curves of the isogeometric shape optimization for three schemes are demonstrated in Fig. 4. The objective functions for all three schemes can converge fast to stable values.

Conclusion:

A unified geometric data is the cornerstone for IGA based structural optimization. Other than traditionally using NURBS to implement shape representation, analysis as well as optimization, this paper constructs a complete framework to capacitate the relevant operations on the basis of unstructured spline models. The necessary technology details to realize shape optimization are explained and the whole process indicates the potentials for combining IGA with unstructured splines to underpin an integrated CAD/CAE/OPT scenario.

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